Example Find these integrals:

(1) $\displaystyle\int \frac{1}{\sqrt{x}(2-\sqrt{x})}\,dx$

(2) $\displaystyle\int_{\pi/4}^{\pi/2} \frac{e^{\cot\theta}}{1-\cos^2(\theta)}\,d\theta$

(3) $\displaystyle\int_{1}^{e} \frac{\ln b}{b}\,db$

(4) $\displaystyle\int \sqrt{\sqrt{y}+2}\,dy$

Solutions

(1) $\displaystyle\int \frac{1}{\sqrt{x}(2-\sqrt{x})}\,dx$

Let

$u = 2-\sqrt{x}$

$du = -\frac{1}{2}x^{-1/2}\,dx$

$du = -\frac{1}{2}\frac{1}{\sqrt{x}}\,dx$

$-2\,du = \frac{1}{\sqrt{x}}\,dx$

$$= \int \frac{-2\,du}{u} = -2\ln|u| + C = \boxed{-2\ln|2-\sqrt{x}| + C}$$

(2) $\displaystyle\int_{\pi/4}^{\pi/2} \frac{e^{\cot\theta}}{\cancel{1-\cos^2(\theta)}\ \sin^2(\theta)}\,d\theta$

Let $u = \cot\theta$

$du = -\underline{\csc^2\theta}\,d\theta$

$du = -\frac{1}{\sin^2\theta}\,d\theta$

$-du = \frac{1}{\sin^2\theta}\,d\theta$

Bounds

$\theta = \frac{\pi}{4}$ $u = \cot\theta = 1$

$\theta = \frac{\pi}{2}$ $u = \cot(\frac{\pi}{2}) = 0$

$$\int_1^0 e^u(-du) = -e^u\Big|_1^0$$

$$= -e^0 - (-e^1) = \boxed{e-1}$$

(3) $\displaystyle\int_1^e \frac{\ln b}{b}\,db$

Let $u = \ln(b)$

$du = \frac{1}{b}\,db$

$b=1$ $u = \ln(1) = 0$

$b=e$ $u = \ln(e) = 1$

$$= \int_0^1 u\,du = \frac{u^2}{2}\Big|_0^1 = \frac{1}{2} - \frac{0}{2} = \boxed{\frac{1}{2}}$$

4 $\displaystyle\int \sqrt{\sqrt{y}+2}\, dy$

<u>Want</u>: $u = \sqrt{y}+2 \Rightarrow du = \frac{1}{2} y^{-1/2} dy$

Not there.

Solve for y.

$u-2 = \sqrt{y}$

$(u-2)^2 = y$

Take d $\quad 2(u-2)du = dy$

$$= \int \sqrt{u}\,\left(\overset{2u-4}{2(u-2)}\right)du$$

$$= \int \left(2u^{3/2} - 4u^{1/2}\right)du$$

$$= \frac{2u^{5/2}}{5/2} - \frac{4u^{3/2}}{3/2} + C$$

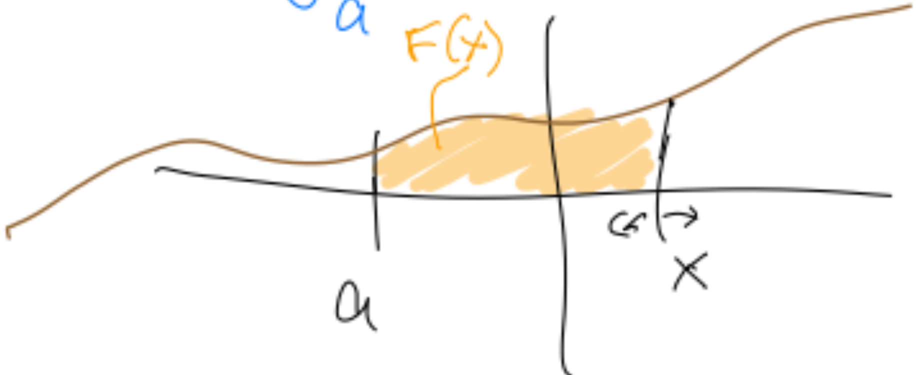
$$= \frac{4}{5}u^{5/2} - \frac{8}{3}u^{3/2} + C$$

$$= \boxed{\frac{4}{5}\left(\sqrt{y}+2\right)^{5/2} - \frac{8}{3}\left(\sqrt{y}+2\right) + C}$$

Second FUNdamental Theorem of Calculus. (2FTC).

If $f(x)$ is a continuous fcn on an interval and a, x are in that interval, then we can define

$$F(x) = \int_a^x f(t)\,dt$$



If we do this, $F'(x)$ exists and

$$F'(x) = f(x).$$

Example of where this is useful:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt$$

important function in statistics.

From 2FTC,

$$(\operatorname{erf}(x))' = \frac{2}{\sqrt{\pi}} e^{-x^2}$$ ← easy to calculate.